Diode Laser Quantum Noise

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DLS LAB

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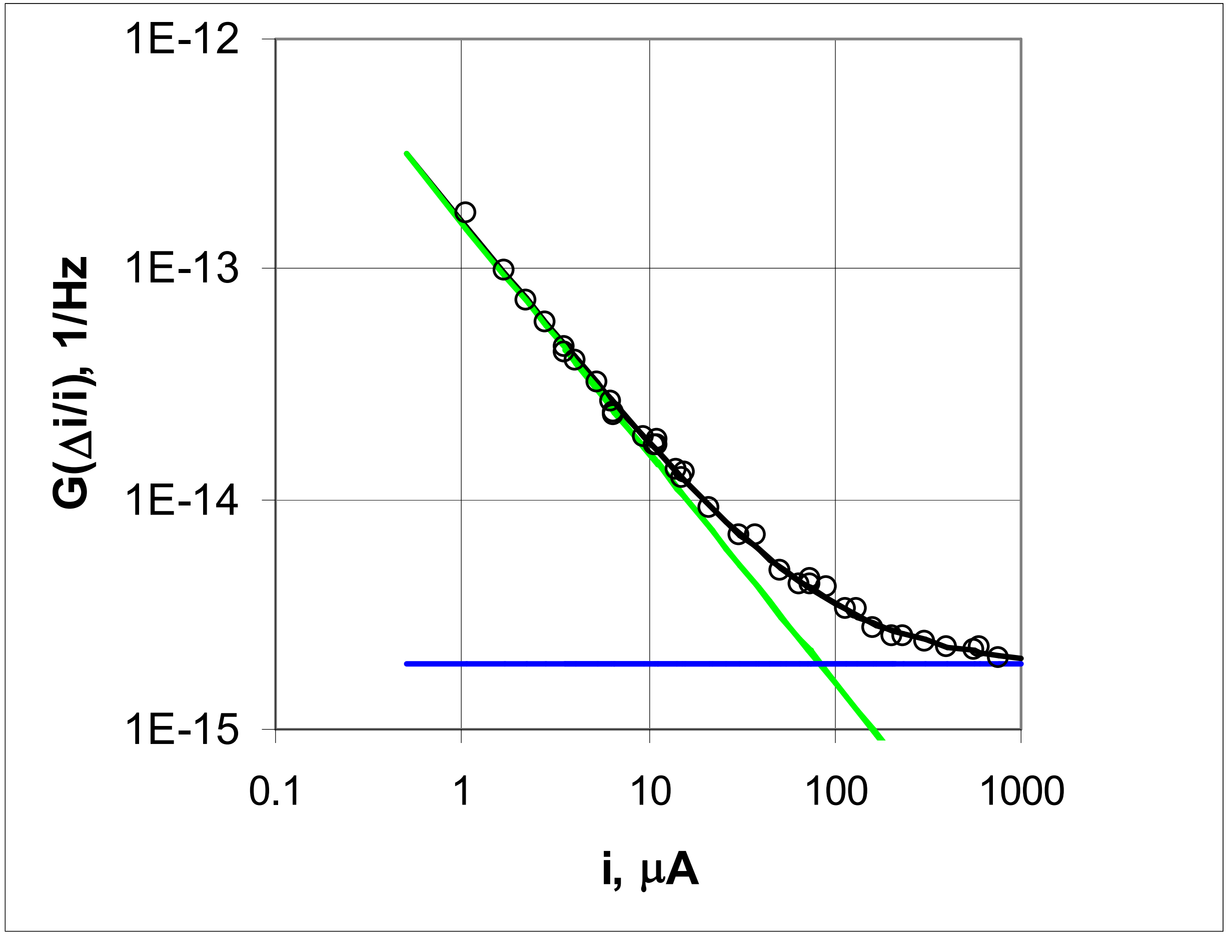
TDLS fundamental limit



Spectral density of relative photocurrent noise – **G(Δi/i)** as function of photocurrent value **i**. Circles correspond to registration of diode laser radiation; green line – theoretical value of photocurrent shot noise - **e/i**. PD was located at different distances from DL to detect small part of DL radiation for the same DL excitation current. ***When all DL radiation is focused on PD, photocurrent will be 20 mA.***

Photocurrent shot noise dominates below 100 μA. Above 100 μA new noise mechanism can be observed (blue line). ***For this mechanism relative noise is constant – noise is generated inside DL. It is DL radiation quantum noise – subject of present paper.***

Back to basic

In quantum physics object don't know is it particle or wave. Our method of measurement determines what parameter of the system we are measuring. To determine what the object parameter is measuring in particular experiment, characteristic experiment dimension has to be compared with the object wave length.

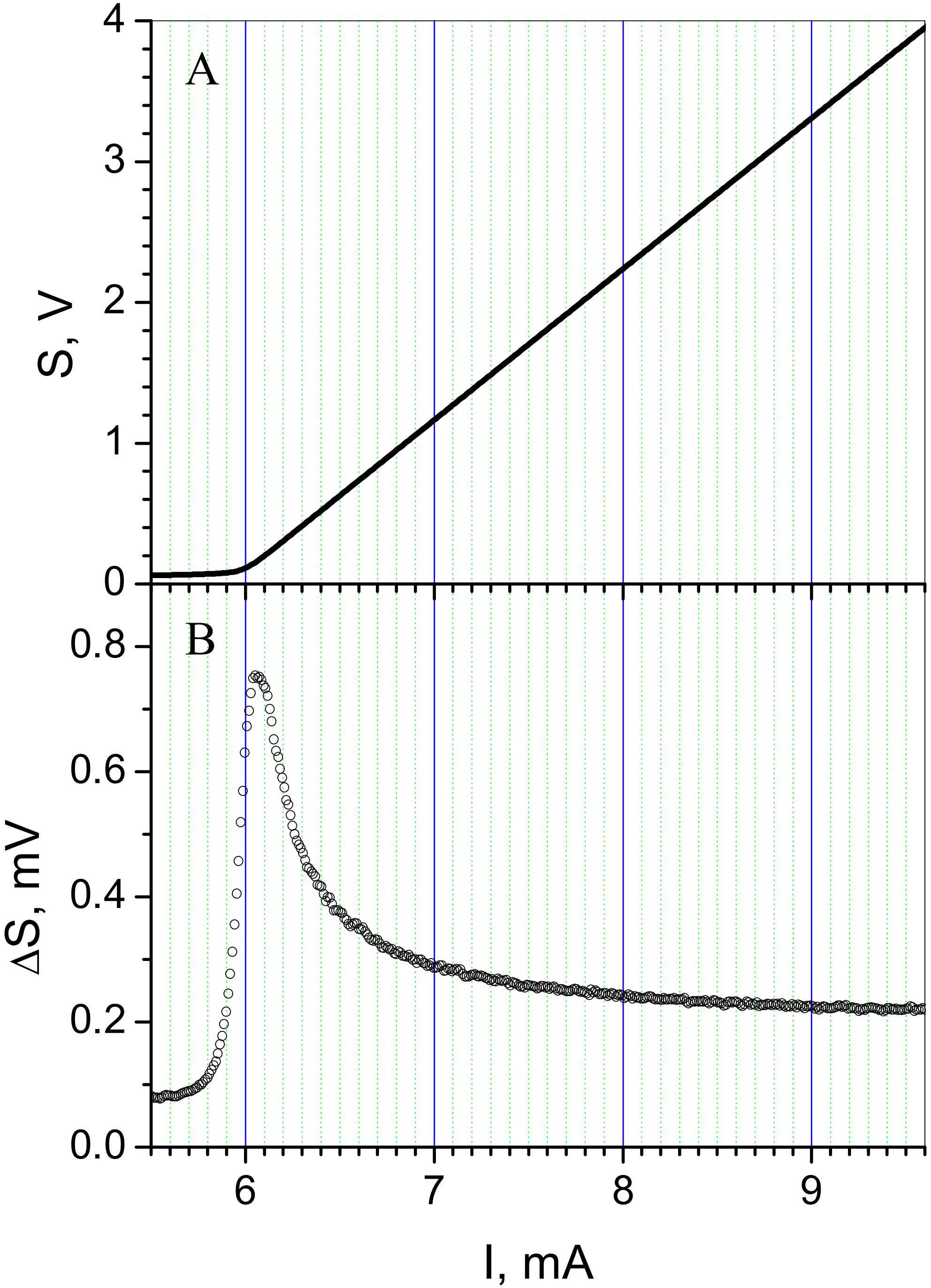
Electron: electron wave length (kinetic energy ~ kT) is equal to 1.2 nm. Electron is wave in quantum well. Electron is particle in photo-detector.

Photon: Photon behavior also depends on its wavelength and characteristic dimension of experiment.

γ quant with energy 124 keV has wavelength 0.01 nm. For atom it is particle. Atom is totally transparent for such γ quant. For nuclei this quant is wave: γ-spectroscopy.

Our spectral range: Both for molecule and electron photon is wave. This results in dipole approximation of radiation theory: DL generation and molecular absorption.

Diode laser emission



Trace molecules detection by TDLS is determined by recorded signal and its noise.

Signal (A) and signal noise (B) dependence vs. excitation current near threshold for one of the DL under investigation (near IR DL).

Signal demonstrates threshold and intermediate area in its vicinity.

Noise asymmetric peak can be observed near threshold. Then noise is achieving constant above threshold and increase (not in this graph) for higher excitation currents.

These dependences are subject of present paper analysis to identify responsible physical mechanisms.

Rate equations

Let us consider rate equations describing radiation generation in DL.

$$\frac{dN_c}{dt} = \chi \frac{I}{e} - gN_c(N_p + 1) + gN_G N_p - \frac{N_c}{\tau_c}$$

$$\frac{dN_p}{dt} = gN_c(N_p + 1) - gN_G N_p - \frac{N_p}{\tau_p}$$

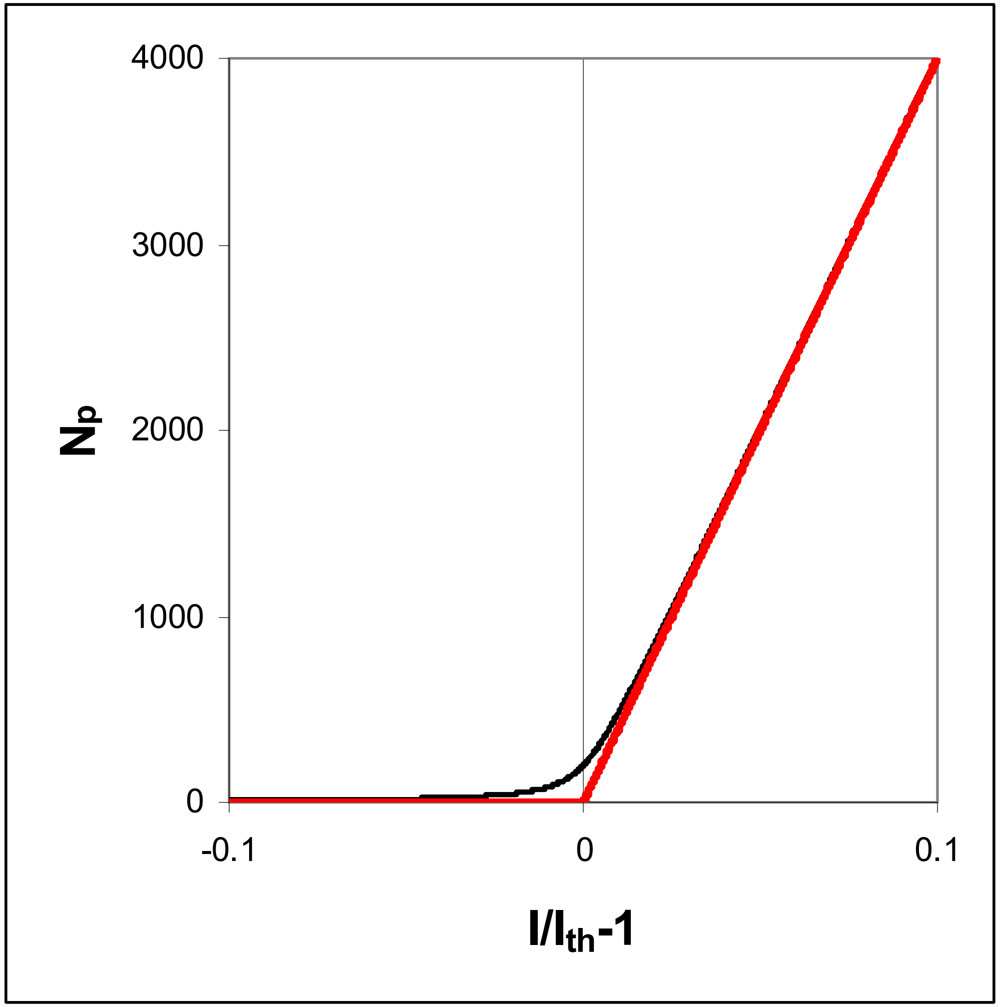
N_c – electrons number in DL active area, N_p – photons number in particular resonator mode, I – excitation current, e – electron charge, χ - quantum efficiency, τ_c – electron life time in energy state interacting with particular resonator mode, g – coefficient describing absorption and stimulated emission, N_G – electrons number when absorption is compensated by stimulated emission, τ_p – photon life time in resonator,

Quantum nature of light is related to presence of spontaneous emission (1 in brackets).

Rate equations stationary solution

$$N_p = N_p(I_{th})^2 \left\{ \frac{1}{2}\left[\frac{I - I_{th}}{I_{th}}\right] + \sqrt{\frac{1}{4}\left[\frac{I - I_{th}}{I_{th}}\right]^2 + \boxed{\frac{1}{N_p(I_{th})^2}\frac{I}{I_{th}}}} \right\}$$

$$N_c = \frac{\tau_c}{\tau_p}\frac{N_p(I_{th})^2 N_p}{(N_p + 1)}$$



Spontaneous emission is responsible for presence of term inside red rectangular.

$$N_p(I_{th}) = \sqrt{N_p(2I_{th})} = \sqrt{\chi \frac{I_{th}}{e}\tau_p}$$

Photons number dependence of excitation current is determined by $N_p(I_{th})$ – photons number at threshold.
Red - classical electromagnetic field (no spontaneous emission, $N_p(I_{th}) \rightarrow \infty$).
Black – $N_p(I_{th})$ = 200 (typical for near IR DL). Below and above threshold black is similar to red. Near threshold - transition area with width reversely proportional to $N_p(I_{th})$.

Rate equations with quantum noises

$$\frac{d\Delta N_c}{dt} = \chi \frac{I + \Delta I}{e} - g\left[N_c + \Delta N_c + \Delta V\right]\left[(N_p + 1) + \Delta N_p + \Delta W\right] +$$

$$+ gN_G\left[N_p + \Delta N_p\right] - \frac{N_c + \Delta N_c}{\tau_c}$$

$$\frac{d\Delta N_p}{dt} = g\left[N_c + \Delta N_c + \Delta V\right]\left[(N_p + 1) + \Delta N_p + \Delta W\right] +$$

$$+ g\Delta F - gN_G\left[N_p + \Delta N_p\right] - \frac{N_p + \Delta N_p}{\tau_p}$$

Four quantum noise mechanisms were introduced and analyzed: ΔI – excitation current shot noise; ΔV – Poison noise of electrons; ΔW – Poison noise of photons; ΔF – quantum noise of electromagnetic field. These noises lead to photons number noise - ΔN_p. As result signal noise will take place.

DL intensity noise

Solution of rate equations with quantum noises (see previous slide) in linear approximation for spectral density G of photons number.

$$G(\Delta N_p) = \frac{N_p(I_{th})^2 (N_p+1)^2}{N_p(I_{th})^2 + (N_p+1)^2} \frac{G(\Delta I)}{I_{th}} + \frac{N_p(I_{th})^2 N_p}{N_p(I_{th})^2 + (N_p+1)^2} G(\Delta W) +$$

$$+ \frac{(N_p+1)^2}{N_p(I_{th})^2 + (N_p+1)^2} \frac{\tau_p}{\tau_c} G(\Delta V) + \frac{\tau_p}{\tau_c} \frac{(N_p+1) + g\tau_c (N_p+1)^2}{N_p(I_{th})^2 + (N_p+1)^2} G(\Delta F)$$

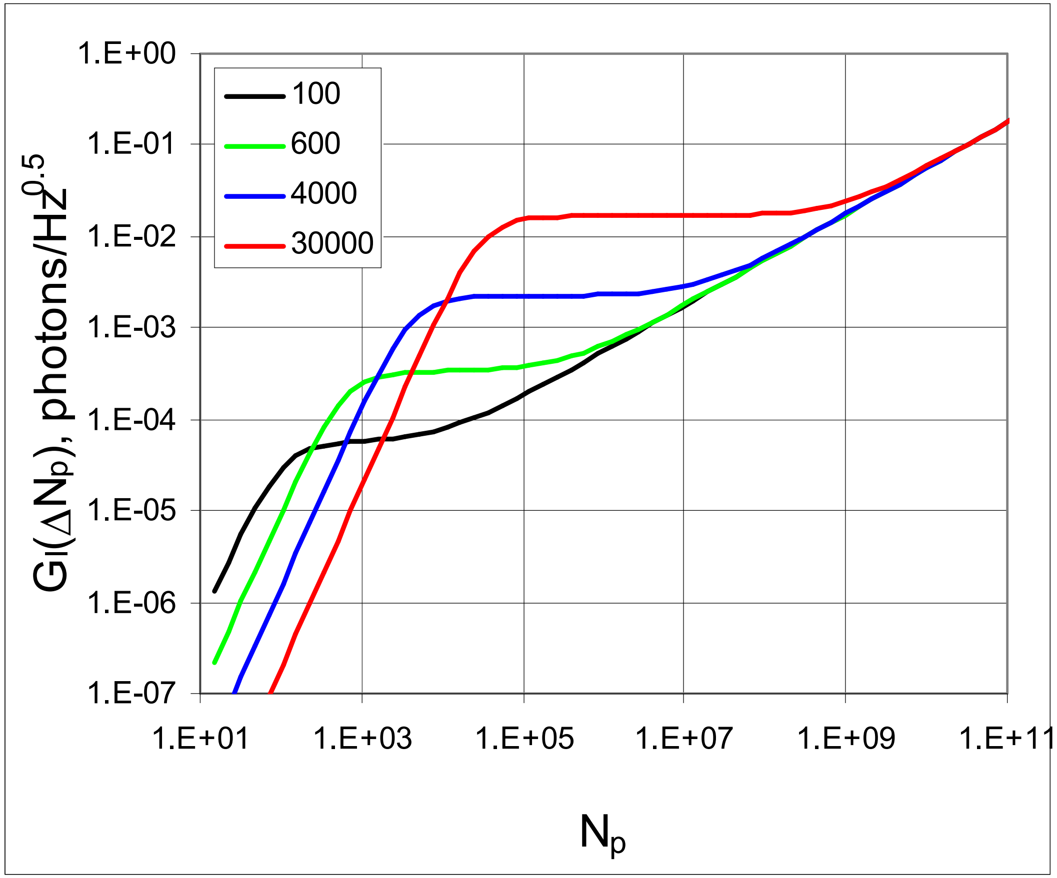
Photons number noise spectral density is determined by spectral density of particular noise mechanism. Two parameters in this equation are known from rate equations stationary solution: N_p and $N_p(I_{th})$. They determine particular noise dependence vs. photons number – N_p with photons number at threshold - $N_p(I_{th})$ as parameter. The rest parameters are subject of determination during model comparison with experiment.

Excitation current shot noise

Electron is particle. Excitation current (I) shot noise spectral density : $G(\Delta I) = \sqrt{eI}$

Using relation obtained above between I and N_p spectral density of photons number noise due to mechanism under consideration can be obtained.

$$G_I(\Delta N_p) = \frac{(N_p+1)^2}{N_p(I_{th})^2 + (N_p+1)^2} \sqrt{\chi \tau_p N_p \left[1 + \frac{N_p(I_{th})^2}{(N_p+1)} \right]}$$



Photons number noise spectral density due to excitation current shot noise as function of N_p. Photons number at threshold - $N_p(I_{th})$ is parameter. Below threshold noise is proportional to N_p^2. Above threshold constant value of noise can be observed being proportional to $N_p(I_{th})$. For very high excitation currents, noise is proportional to $N_p^{0.5}$ and doesn't depend on $N_p(I_{th})$. For real DLs only beginning of this dependence part can be observed.

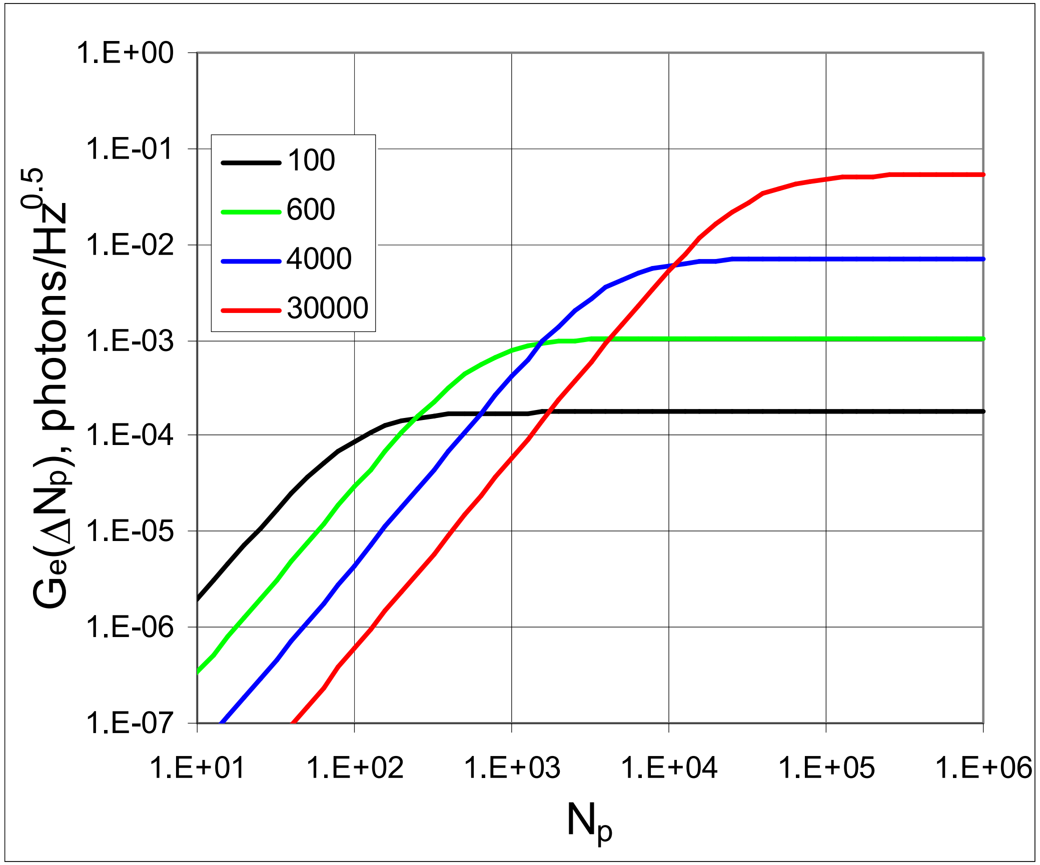
Electrons Poisson noise

Electron is particle – Poisson noise of electrons number N_c in DL active area. Its spectral density is determined by std(ΔN_c) = $N_c^{0.5}$ and electrons life time - τ_c.

$$G(\Delta V) = \sqrt{2\tau_c N_c}$$

Using known relation between N_c and N_p noise spectral density due to electrons Poisson noise:

$$G_e(\Delta N_p) = \frac{\sqrt{2\tau_p}(N_p+1)^{3/2} N_p(I_{th})\sqrt{N_p}}{N_p(I_{th})^2 + (N_p+1)^2}$$

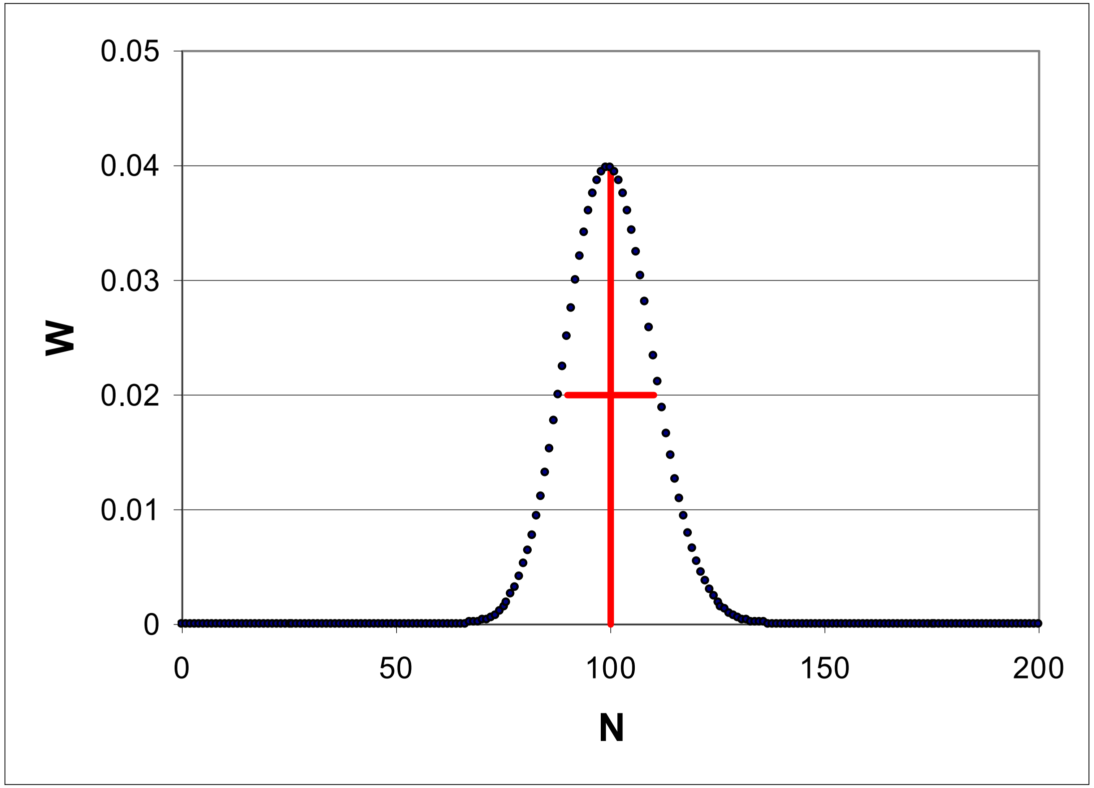


Photons number noise spectral density due to electrons Poisson noise as function of N_p. Photons number at threshold - $N_p(I_{th})$ is parameter.
Below threshold noise is proportional to N_p^2. Above threshold constant value of noise can be observed = $(2\tau_p)^{0.5}N_p(I_{th})$.

Coherent states of light

Coherent state Z plays important role in Quantum optics. Это есть распределение Пуассона.

$$\left|Z\right\rangle = \exp\left(-\frac{1}{2}\left|Z\right|^2\right)\sum_{N=0}^{\infty}\frac{Z^N}{\sqrt{N!}}\left|N\right\rangle$$



W – probability to find N photons in particular mode of DL resonator. This distribution has maximum for N = Z.
The distribution has width – √Z (uncertainty principle – quantum nature of light). Coherent state has smallest uncertainty with respect to other states of light.

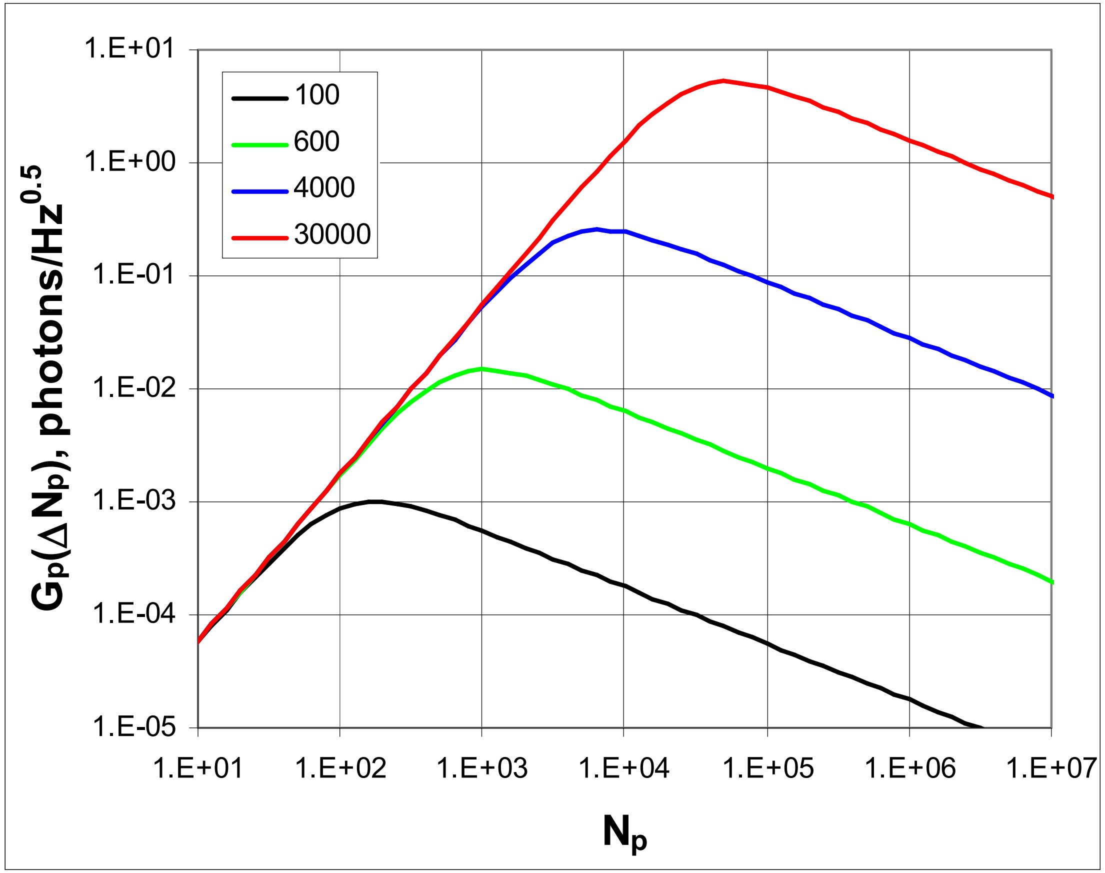
In present case Z = 100 photons in resonator mode, uncertainty - √Z = 10 photons (red lines).

Photons Poisson noise

Coherent state uncertainty will lead to photon Poisson noise. Its spectral density is determined by std(ΔN_p) = $N_p^{0.5}$ and electrons life time - τ_p.

$$G(\Delta W) = \sqrt{2\tau_p N_p}$$

$$G_p(N_p) = \frac{N_p(I_{th})^2 N_p \sqrt{2\tau_p N_p}}{N_p(I_{th})^2 + (N_p + 1)^2}$$



Photons number noise spectral density due to photons Poisson noise as function of N_p. Photons number at threshold - $N_p(I_{th})$ is parameter. Below threshold noise is proportional to $N_p^{1.5}$ and does not depend on $N_p(I_{th})$. Noise maximum can be observed near threshold. Above threshold $N_p^{0.5}$ noise dependence takes place.

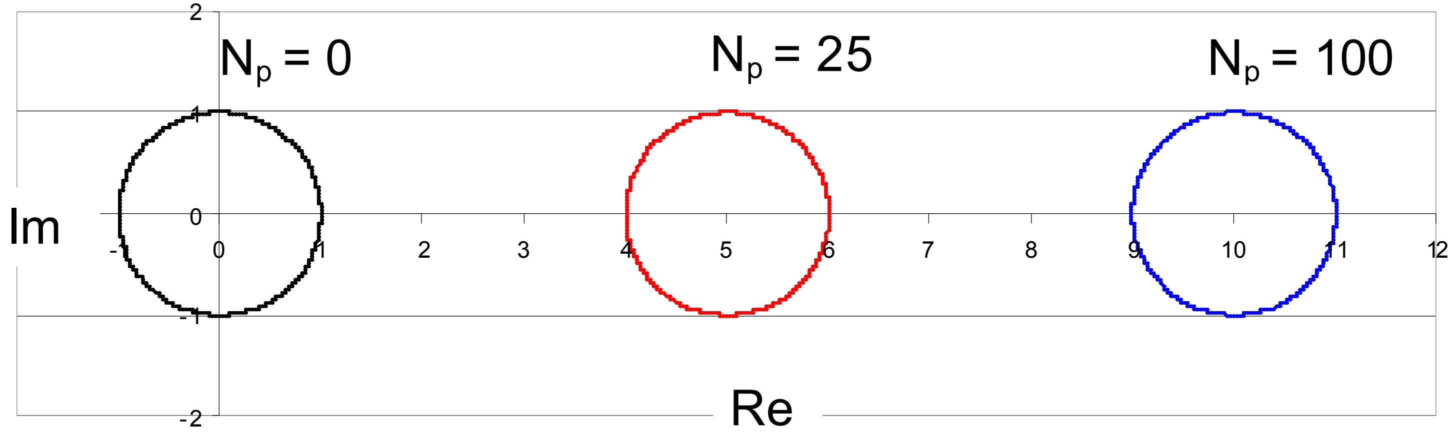
The higher threshold current the higher noise.

Quantum Electromagnetic field

Due to quantum nature of light both intensity and phase of light emission have uncertainty. Matrix element of photon emission:

$$A = \left\langle N_p + 1 \middle| \hat{c}^+ \middle| N_p \right\rangle = \sqrt{N_p} + \exp(i\varphi)$$

Vector presentation of photon emission matrix element for different photons numbers.



Photon emission probability for one electron in DL active area.

$$AA^* = N_p + 2\sqrt{N_p}\cos(\varphi) + 1$$

For one electron nothing new will be observed. Averaging over realizations will lead to well-known emission probability - N_p+1.

Electromagnetic field Quantum noise

Presence of N_c electrons in DL active area will lead to interference of light emitted by each electron in DL resonator mode and presence of electromagnetic field noise:

$$\Delta F(t) = 2\sqrt{N_p}\sum_{n=1}^{N_c}\cos(\varphi_n)$$

Std of this noise can be calculated straightforward.

$$std\big(\Delta F(t)\big) = \sqrt{2N_pN_c} = N_p\sqrt{2\frac{\tau_c}{\tau_p}\frac{N_p(2I_{th})}{(N_p+1)}}$$

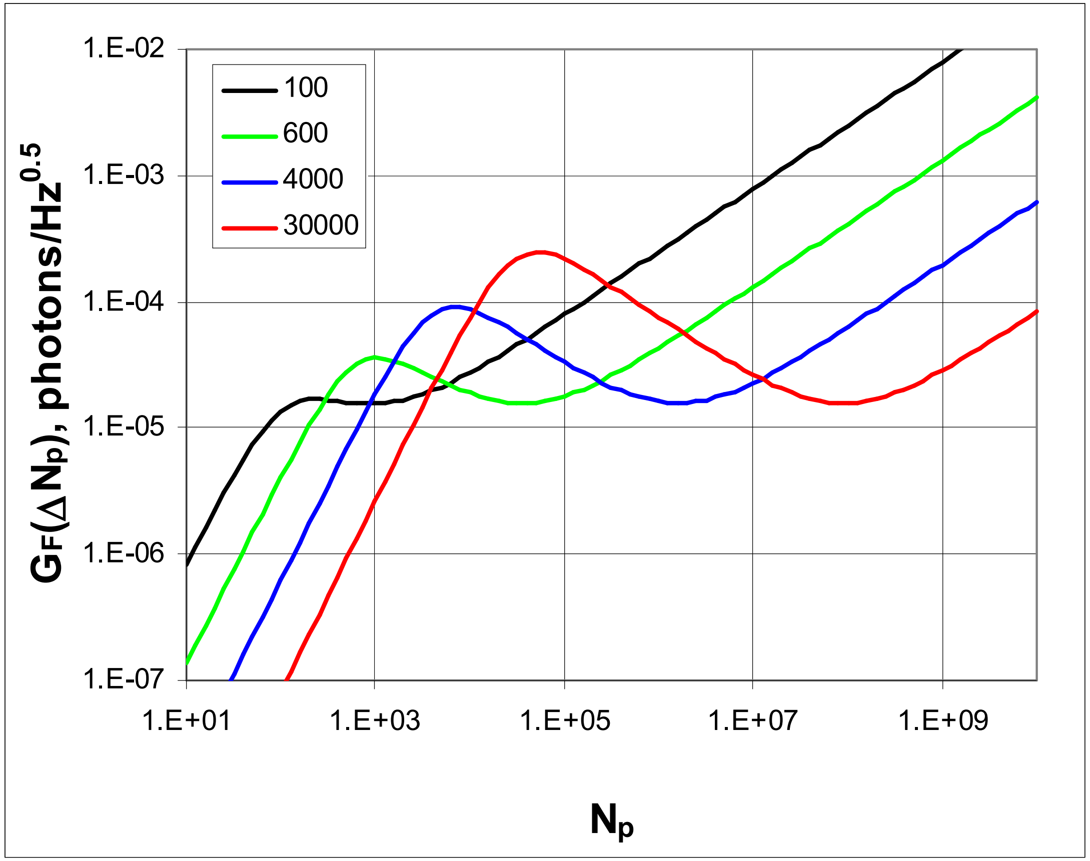
The noise spectral density can be determined using electrons life time τ_c:

$$G\big(\Delta F(t)\big) = 2N_p\tau_c\sqrt{\frac{1}{\tau_p}\frac{N_p(I_{th})^2}{(N_p+1)}}$$

Field quantum noise

Based of analysis performed above noise spectral density due to electromagnetic field quantum noise can be determined

$$G_F\left(\Delta N_p\right) = 2\frac{1+g\tau_c(N_p+1)}{N_p(I_{th})^2+(N_p+1)^2}N_p\sqrt{\tau_p N_p(I_{th})^2(N_p+1)}$$

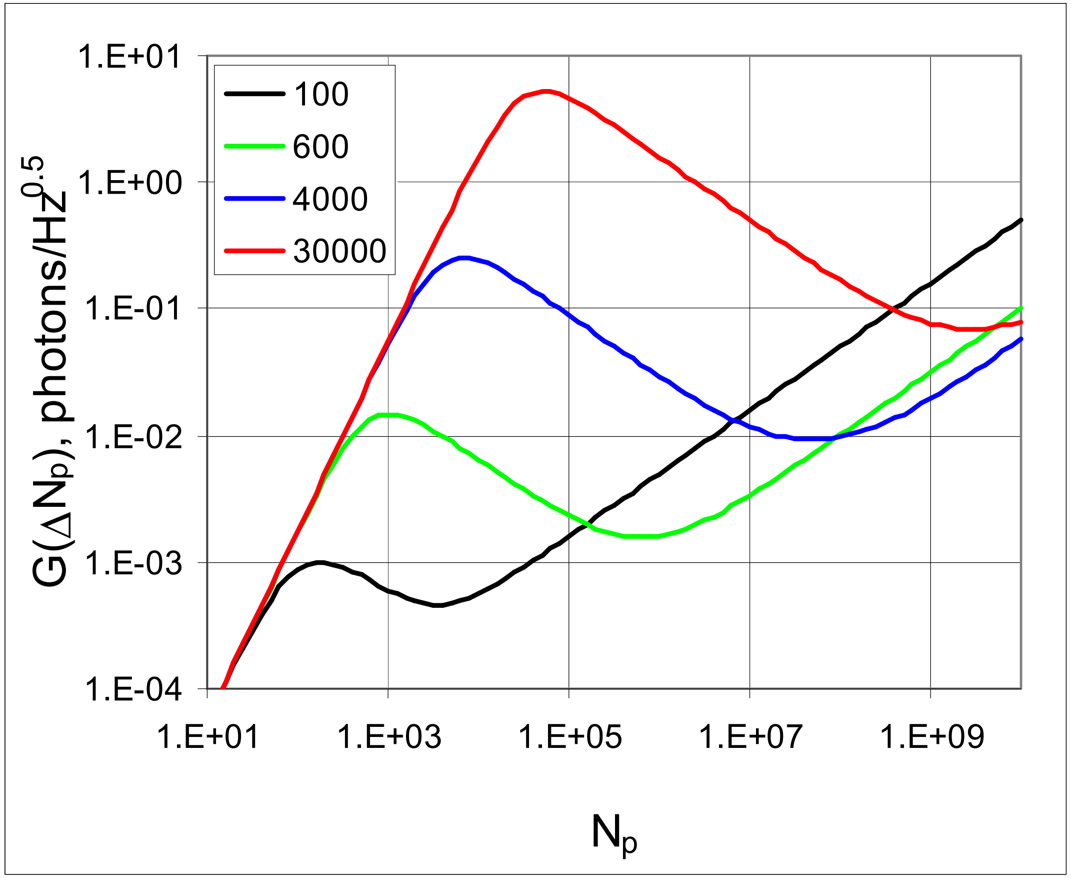


Photons number noise spectral density due to electromagnetic field quantum noise as function of N_p. Photons number at threshold - $N_p(I_{th})$ is parameter.
This picture can be considered only as qualitative because two parameters $N_p(I_{th})$ and $g\tau_c$ are not independent.
For high excitation currents this mechanism is negligible for high $N_p(I_{th})$. The closer electromagnetic field to classical one the less influence of the noise mechanism under consideration.

Conclusion

Four DL noise mechanism were introduced and analyzed:

1. Excitation current shot noise.
2. Electrons Poisson noise.
3. Photons Poisson noise.
4. Quantum noise of electromagnetic field.



Final photons number noise spectral density as function of N_p taking into account noise mechanisms under consideration. Photons number at threshold - $N_p(I_{th})$ is parameter.

Dominant noise depends on DL in use and experiment set up.

Based on noises analysis performed TDLS operation mode can be optimized.

Model developed will be compared with experiment in C1 and D1.